

# 

### Semester Two Examination, 2018

### Question/Answer booklet

# MATHEMATICS

**SOLUTIONS**

**METHODS**

**UNITS 3 AND 4**

## Section One:

## Calculator-free

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Student number: In figures |  |  |  |  |  |  |  |  |  |  |

In words

Your name

## Time allowed for this section

Reading time before commencing work: five minutes

Working time: fifty minutes

## Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer booklet

Formula sheet

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,  
correction fluid/tape, eraser, ruler, highlighters

Special items: nil

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Section | Number of questions available | Number of questions to be answered | Working  time (minutes) | Marks available | Percentage of examination |
| Section One:  Calculator-free | 8 | 8 | 50 | 52 | 35 |
| Section Two:  Calculator-assumed | 13 | 13 | 100 | 98 | 65 |
|  | | |  | **Total** | 100 |

## Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Supplementary pages for the use of planning/continuing your answer to a question  
have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

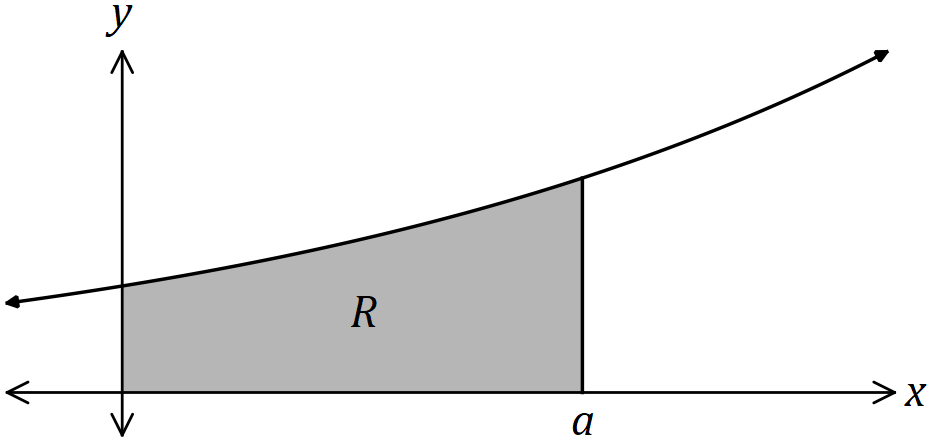
Section One: Calculator-free 35% (52 Marks)

This section has**eight (****8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 (6 marks)

The shaded region , shown on the graph below, is bounded by the curve and the lines and .



(a) Determine the area of in terms of . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
|  writes correct integral  ✓ antidifferentiates correctly   substitutes and simplifies |

(b) Determine, in simplest form, the value of for which the area of is square units.

(3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ isolates term   uses logs to obtain expression for   simplifies |

Question 2 (5 marks)

(a) Simplify . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ expresses as powers of log bases   simplifies |

(b) Solve the equation . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ writes both sides as single logs   factorises quadratic   identifies just one solution |

Question 3 (7 marks)

The graph of has one local minimum and one local maximum.

(a) Determine the ­­­­-coordinates of the stationary points of the graph. (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses product rule correctly   factorises derivative   states -coordinates |

(b) Use the second derivative test to determine which of the points from (a) is a local maximum and state the coordinates of this point. (4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ second derivative using product rule   substitutes an -value   interprets sign of second derivative   deduces required -value and states coordinates |

Question 4 (6 marks)

The random variable has probability density function

(a) Determine the value of the constant . (4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ writes integral with correct limits   integrates correctly   equates integral to   correct value of |

(b) Write down the cumulative distribution function for and hence determine . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct   correct probability |

Question 5 (6 marks)

(a) Determine the anti-derivative of . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ writes in form   correct integral and constant |

(b) Determine when (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses product rule correctly   differentiates log term correctly |

(c) Evaluate . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ antiderivative   evaluates correctly |

Question 6 (8 marks)

Right-triangle has vertices and where and .

The straight line from to passes through the point .

(a) Sketch a diagram to show this information. (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ triangle with point, and |

(b) Show that the area of the triangle is . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses ratio of sides   expresses in terms of   uses area of triangle |

(c) Determine the value of that minimises the area of the triangle and state the minimum area. (4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses quotient rule correctly   equates numerator equal to   solves for   calculates area |

Question 7 (6 marks)

The time, years, to repay a loan of at interest with monthly repayments of dollars can be approximated by

(a) Determine the time to repay the loan when the monthly repayment is , simplifying your answer. (1 mark)

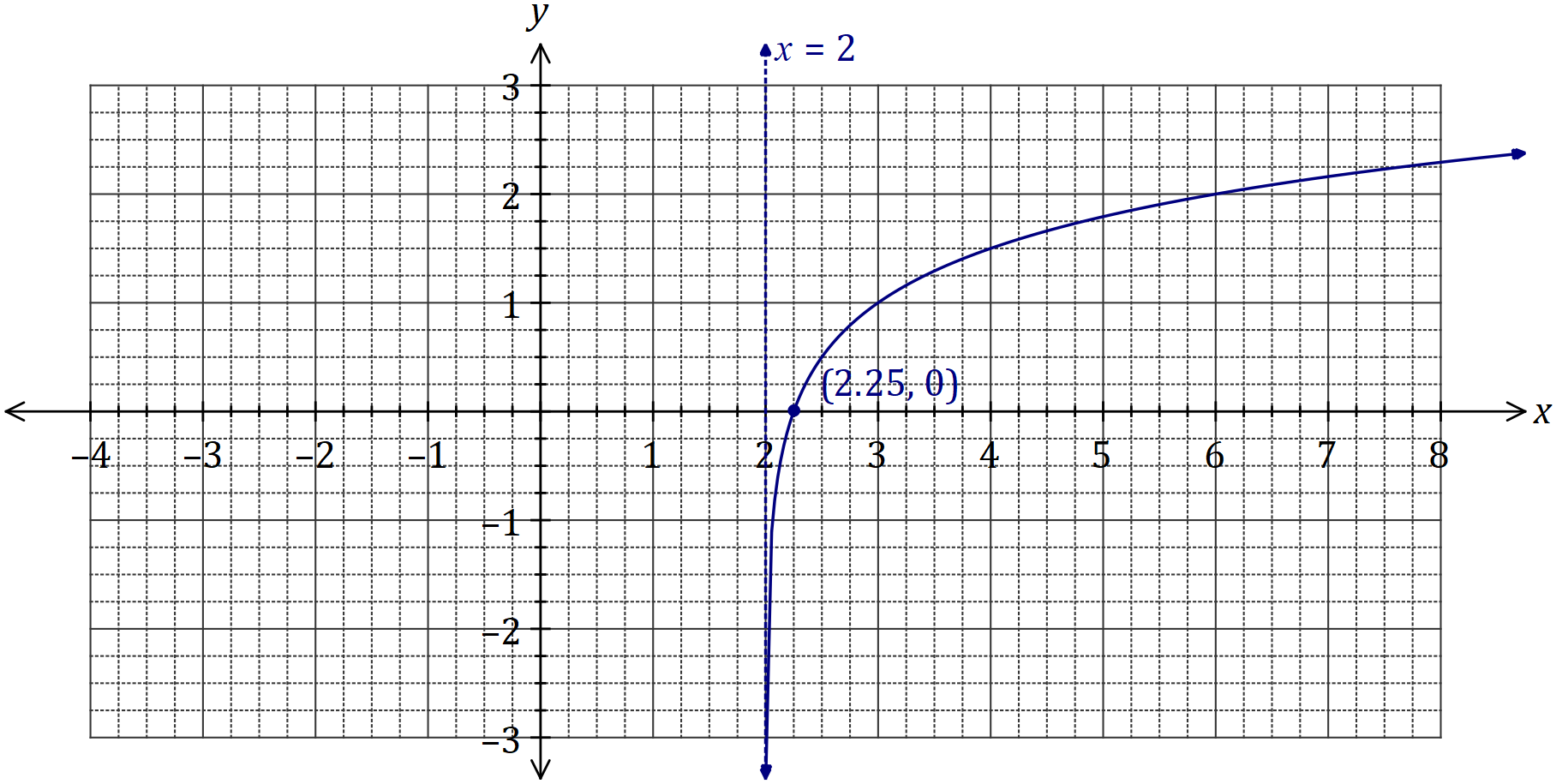
|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ substitutes and simplifies |

(b) Use the increments formula to estimate the time saved in repaying the loan if the monthly repayment of is increased by . (5 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
|  uses log laws to simplify   correct derivative   evaluates derivative  ✓ indicates value of   uses increments formula and states time saved |

Question 8 (8 marks)

(a) Sketch the graph of on the axes below, clearly showing the location of all asymptotes and axes intercepts. (3 marks)



|  |
| --- |
| **Solution** |
| See graph |
| **Specific behaviours** |
| ✓ asymptote   root at   curve, through and |

(b) Determine the coordinates of the -intercept of the graph of .

(2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ simplifies log term to   states coordinates of intercept |

(c) The graph of , where , passes through . Determine the coordinates of the root of the graph. (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ forms quadratic equation   solves for   states coordinates of root |

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

© 2018 WA Exam Papers. Kennedy Baptist College has a non-exclusive licence to copy and communicate this document for non-commercial, educational use within the school. No other copying, communication or use is permitted without the express written permission of WA Exam Papers. SN245-125-2.